THE DYNAMICS OF NETWORK-EFFECTS IN TWO-SIDED AND MULTI-SIDED MARKETS: AN AGENT-BASED APPROACH

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ABSTRACT

Two-sided and multi-sided markets and the dynamics of their network-effects have become an active research area in recent years. In this paper a general formulation of an agent-based duopoly-model of a two-sided market is presented that seems very promising to study and to analyze under which starting conditions two-sided markets tend (just because of the dynamics of the involved network-effects) to a winner-takes-all-situation and under which starting conditions both platforms can survive within the market. The results of the simulations of this model can be used for analysing real existing two-sided (and multi-sided) markets.

Keywords: Two-sided markets, multi-sided markets, network-effects, agent-based model

INTRODUCTION

Many modern markets try to tie together two or more distinct groups of individuals (users) in a network whereas the individuals of each group are interested in interacting with the individuals of the other group. These markets are called ‘two-sided’ or ‘multi-sided markets’ (depending on the number of different groups) and can roughly be defined as “markets in which one or several platforms enable interaction between end-users, and try to get the two (or multiple) sides ‘on board’ by appropriately charging each side” (Rochet/Tirole 2005).

If someone thinks about that definition, a lot of markets rapidly come into one’s mind: Newspapers for example have to attract readers and advertisers, videogame platforms have to attract consumers and software developers, TV networks have to attract viewers and advertisers, credit cards have to attract cardholders and merchants and so on (Rochet/Tirole 2005; Evans 2003). The actual importance of markets like these is quite enormous, since these markets have redefined and changed the global business landscape rapidly and fundamentally in the last few decades (Eisenmann/Parker/Van Alstyne 2006).

One interesting thing of two-sided and multi-sided markets is that frequently they cannot be analyzed by means of the traditional and well-known economic rules. Many of them are not working correctly in these markets any more and so the use of these traditional rules can sometimes lead to severe mistakes (and losses) (Wright 2004).

The key issues to understand the way how two-sided and multi-sided markets work seem to be the so-called ‘same-side’ and ‘cross-side’ network effects. Simply spoken, network effects

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in general describe the possibility that the derived utility for a user by the consumption of a certain good increases with the number of other users consuming the good as well (Katz/Shapiro 1985; Katz/Shapiro 1986; Tirole 2002). Since in two-sided and multi-sided markets (where in a simple model the ‘good’ is just to join or stay at a platform) two or more distinct groups of users are involved, two kinds of network effects can be distinguished: While same-side network-effects describe the network effects within each group of users, cross-side network effects describe the network effects between the two or more groups of users (respectively between the individuals of the two or more market sides).

RELATED WORK

Since two-sided and multi-sided markets frequently cannot be analyzed by means of the well-known economic rules, the implication for academic research seems to be quite clear: It is necessary to find adapted models to understand the nature of two-sided and multi-sided markets and to understand how the mechanisms of these markets work. In order to do so, especially in the last few years a lot of research has been done in this field in various directions. In the next paragraphs two major research directions are briefly introduced.

In a first important research field the new mechanisms of adding value are explored: While in traditional markets value moves from the left to the right side (where to the left side is cost and to the right side is revenue), in two-sided and multi-sided markets, cost and revenue are both to the left and the right, because the platform has to incur costs while serving both groups, but can also collect revenue from each side (Eisenmann/Parker/Van Alstyne 2006). As a result, the traditional model of the linear value-added chain is not very suitable for two-sided and multi-sided markets and has to give way for models with value-added helices, which are more qualified for describing the feedback effects and dependencies between the market sides (Dietl/Frank/Royer 2006).

A second research direction deals with the analysis of the optimal price setting in two-sided and multi-sided markets: In traditional (one-sided) markets firms have to choose the price level by which the sold quantity normally can be influenced (assuming a typically sloped demand function). Opposed to that, in two-sided or multi-sided markets firms (respectively platforms) must choose a price structure, which means that they have to find a price level for each market side and not just a general price level (Rochet/Tirole 2003). That fact and the mutual dependence of the different prices on each market side yield to the problem that setting prices in two-sided and multi-sided markets is a highly complex issue. The same holds for the analysis of the (dynamic) price competition between platforms and their pricing behaviour in different model environments (e.g. in models with product differentiation). General models of two-sided and multi-sided markets dealing with price competition and pricing behaviour can be found in Armstrong (2005) and in Rochet/Tirole (2005). Further (and sometimes more specific) models can be found (for example) in Armstrong/Wright (2007), Caillaud/Jullien (2001), Caillaud/Jullien (2003), Parker/Van Alstyne (2005), Rochet/Tirole (2002), Rochet/Tirole (2003) and Yoo/Choudhary/Mukhopadhyay (2002). Some empirical results concerning pricing behaviour (among other areas) in two-sided and multi-sided markets can be seen in Evans (2003).

RESEARCH QUESTION

Although a lot of efforts have been made to understand the way how two-sided and multi-sided markets work and although this research topic is being processed intensively at the moment, there is still a lot to do in order to understand the dynamics of the involved network effects. While analytical approaches in the field of two-sided and multi-sided markets very often have to face the general problem of an exploding complexity of the models, an agent-based approach seems to be an ideal method in this research area. It has to be mentioned that some authors have already used the agent-based approach in the field of two-sided and multi-sided markets, but with a different focus (e.g. Chen/Mäikö 2006 and Kabadjova/Tsang/Krause 2006). However, it seems that the use of the agent-based method can make it especially possible to understand this described key issue, namely the dynamics of the network effects, more accurately and more generally.

To be more precise, the agent-based approach allows to answer the research question, under which starting conditions (for example the number of individuals/agents at each platform on each side, the size of the weighting-factors for the size of the same-side and cross-side-effects, the dullness of the decisions etc.) two-sided and multi-sided markets tend (just because of the dynamics of the network-effects) to a winner-takes-all-situation (where just one platform survives) and under which starting conditions two or more platforms can survive within the market. After simulating this model the results can probably be used to analyse real existing two-sided (and multi-sided) markets.

To simplify things, only two-sided markets are considered in the model that is introduced in this progress report. The more complex multi-sided markets are likely to be considered in future research projects. However, the presented model can easily be expanded to incorporate more than two market-sides and therefore to consider these multi-sided markets as well.

AGENT-BASED MODEL

In the following subsections a simple but general agent-based model of a two-sided market is introduced, where all agents (respectively individuals) on each market side are confronted with a duopoly (meaning two platforms). Though these agents cannot change their market side (respectively their group), these agents have to decide in each period, which of the two platforms they should choose on their market side (whereas just one platform can be chosen at any time). In this model, the mentioned agents' decisions are made according to the agents' derived net-utility of each platform (a concept which is for example also used by Rochet/Tirole (2005)) which depends on the demanded prices set by the platforms, on the path-depending size of the number of other agents at the same market side (at a specific platform) and the number of other agents at the other market side (also at that specific platform), each weighted with factors that determine the size of the same-side and cross-side network effects.

Individuals, Groups and Platforms

Let's assume there exists a two-sided market with two groups of individuals (two market sides) denoted by $L, M \in \Gamma$ (whereas $L \neq M$). The individuals $l \in L$ of any of these two groups $L \in \Gamma$ are interested in interacting with the individuals $m \in M$ of the other group $M \in \Gamma$, $L \neq M$.

Let's assume further on that there exist two platforms $i, j \in P$. Each platform can help to bring together individuals of the two groups at certain points of time if individuals of both groups decide to join this platform or remain at this platform at exactly those points of time.

It is assumed that an individual stays exactly at one of the two platforms at a certain point of time, but that he or she is in general able to change the chosen platform at any of those points of time without switching costs (Varian 2003). Therefore let $l_t^i \in L_t^i$ denote an individual $l \in L$, $L \in \Gamma$ that stays at platform $i \in P$ at time $t \in T$. Because each individual stays exactly at one of the two platforms at any point of time $t \in T$, because no individual is able to change his or her group and because neither existing individuals can disappear nor new individuals can appear in the model, it follows that (whereas $i, j \in P$, $i \neq j$, $L \in \Gamma$ and $t \in T$):[1]

$$(1) \qquad \left|L_t^i\right| + \left|L_t^j\right| = \left|L_t\right| = \left|L_0\right| = \left|L\right| .$$

Let further denote $N(L_t^i)$ as the number of individuals of a group $L \in \Gamma$ that stay at platform i at time t, which means that $N(L_t^i)$ is defined as (whereas $i \in P$, $L \in \Gamma$, $t \in T$):

$$(2) \qquad N(L_t^i) = \left|L_t^i\right| .$$

Net-utility functions

First of all it is necessary to mention that all individuals in the model have static expectations, which means that they expect all observable variables at $t \in T$ to stay the same in the next period $t+1 \in T$.

Let $\alpha^{l,M}$ denote a utility-weighting-factor for a certain individual $l \in L$, $L \in \Gamma$ that determines how much utility a certain group of individuals $M \in \Gamma$ creates for this individual l. Let further $p_t^{i,L}$ denote the price (set by platform $i \in P$) that individuals of the group $L \in \Gamma$ have to pay at time $t \in T$ if they want to join the platform $i \in P$ or remain at this platform at time $t \in T$.

The net-utility of platform $j \in P$ for an individual $l \in L$, $L \in \Gamma$ that stays at platform $i \in P$ at time $t \in T$ is denoted by $u^{l_t^i}(j)$ and depends on the price set by the platform $j \in P$ that individuals of the group $L \in \Gamma$ have to pay at $t \in T$. Further on, this net-utility depends on the gross-utility that individuals of group $M \in \Gamma$ (whereas $L \neq M$) generate for this individual at time $t \in T$ ('cross-side effect') and the gross-utility that individuals of group $L \in \Gamma$ generate for this individual at time $t \in T$ ('same-side effect').

More specifically spoken, each individual $l \in L$, $L \in \Gamma$ that stays at platform $i \in P$ at time $t \in T$ calculates two net-utilities: one for platform $i \in P$ (where the individual stays at time $t \in T$) and one for the other platform $j \in P$ (whereas $i \neq j$). Concretely that means that for individual $l \in L$, $L \in \Gamma$ that stays at platform $i \in P$ at time $t \in T$ the net-utility of this platform $i \in P$ is then given by (whereas $L, M \in \Gamma$ and $L \neq M$):

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[1] Mathematical remark: If A is a set, $|A|$ denotes the number of elements that are contained in A.

$$(3) \qquad u^{l_t^i}(i) = \underbrace{\alpha^{l,L}(N(L_t^i)-1)}_{\substack{\text{gross utility generated}\\ \text{by group } L\\ (\text{same-side effect})}} + \underbrace{\alpha^{l,M} N(M_t^i)}_{\substack{\text{gross utility generated}\\ \text{by group } M\\ (\text{cross-side effect})}} - \underbrace{p_t^{i,L}}_{\text{price}} .$$

In a similar way for the same individual at time $t \in T$ the net-utility of the other platform $j \in P$ is then given by (whereas $i \neq j$, $L, M \in \Gamma$ and $L \neq M$):

$$(4) \qquad u^{l_t^i}(j) = \underbrace{\alpha^{l,L} N(L_t^j)}_{\substack{\text{gross utility generated}\\ \text{by group } L\\ (\text{same-side effect})}} + \underbrace{\alpha^{l,M} N(M_t^j)}_{\substack{\text{gross utility generated}\\ \text{by group } M\\ (\text{cross-side effect})}} - \underbrace{p_t^{j,L}}_{\text{price}} .$$

According to the static expectations of all individuals these two derived utilities are used to determine the behaviour of the individuals at the beginning of the next period $t+1 \in T$. The concrete behaviour of the individuals is explained in a later subsection.

Behaviour of the platforms

To simplify the model, it is assumed that both platforms $i, j \in P$ (whereas $i \neq j$) set identical price structures over time. That means that at all periods of time $t \in T$ both platforms charge the same price p on the same market side $L \in \Gamma$. Mathematically this means that

$$(5) \qquad \forall t \in T : p_t^{i,L} \equiv p_t^{j,L}$$

or equivalently

$$(5a) \qquad \forall t \in T : p_t^{i,L} - p_t^{j,L} = 0 .$$

Although this assumption can be identified as being drastic in some respects, there are at least two reasons why this assumption makes sense in that model: Firstly, this assumption allows to concentrate on the pure dynamic effects that are caused by the same-side and cross-side effects (see next subsection) and secondly, in many two-sided markets price competition and elastic prices don't allow platforms to charge any price from the individuals – which means that they have to finance their business by means of different ways (e.g. by advertising for other companies).[2] Mathematically this means that

$$(5b) \qquad \forall t \in T : p_t^{i,L} = p_t^{j,L} = 0 .$$

This is completely compatible to the assumption made in equation (5). Nevertheless it has to be mentioned that this assumption is very radical, since there is no possibility for platforms to act and react in the model. They are in some sense completely passive, which can be criticised as being unrealistic. However in later research projects it is planned to weaken this assumption to face these claims.

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[2] Of course, if a two-sided market finances this platform by means of advertising for other companies, this platform can in fact also be seen as a three-sided (respectively multi-sided) market.

Behaviour of the individuals

Let's assume $h(x-y)$ to be a classical Heaviside step function (whereas x and y are real numbers):

$$(6) \qquad h(x-y)=\begin{cases}0 & if \quad (x-y)\leq 0\\ 1 & if \quad (x-y)>0\end{cases}.$$

Let's assume further that z is a monotonically increasing function whose image set is assumed to be bounded such that $0\leq z\leq 1$. The probability that an individual $l\in L$, $L\in\Gamma$ (that stays at platform $i\in P$ at time $t\in T$) changes to the other platform $j\in P$ (whereas $i\neq j$) at the beginning of time $t+1\in T$ is then assumed to be

$$(7) \qquad w_{t+1}^{l_t^i}=w_{t+1}^{l_t^i}(\underbrace{u_t^{l_t^i}(j)-u_t^{l_t^i}(i)}_{utility\ difference})=h_{t+1}^{l_t^i}(\underbrace{u_t^{l_t^i}(j)-u_t^{l_t^i}(i)}_{utility\ difference})\cdot z_{t+1}^{l_t^i}(\underbrace{u_t^{l_t^i}(j)-u_t^{l_t^i}(i)}_{utility\ difference}).$$

In this equation the term $h_{t+1}^{l_t^i}(utility\ difference)$ secures the rationality of the individuals ('rationality term'): If the utility difference[3] is negative (which means that there is no incentive for an individual to change the platform) this term $h_{t+1}^{l_t^i}(utility\ difference)$ becomes 0. As a result the whole probability that an individual changes to the other platform $w_{t+1}^{l_t^i}(utility\ difference)$ becomes 0 as well. If however the utility difference is positive (which means that there is an incentive for an individual to change to the other platform) then the term $h_{t+1}^{l_t^i}(utility\ difference)$ becomes 1 and the probability that an individual changes to the other platform is just depending on the term $z_{t+1}^{l_t^i}(utility\ difference)$. $z_{t+1}^{l_t^i}(utility\ difference)$ is kind of a 'dullness term' (respectively 'dullness function') which ensures that if the utility difference is (although positive) only relatively small, the probability that an individual switches to the other platform $w_{t+1}^{l_t^i}(utility\ difference)$ is also relatively small. If however the utility difference is relatively high, the term $z_{t+1}^{l_t^i}(utility\ difference)$ ensures that the probability that an individual changes to the other platform $w_{t+1}^{l_t^i}(utility\ difference)$ is relatively high as well.

In this general model z is not specified exactly. In concrete agent-based simulations several specifications of the term (or function) z are possible, as long as z is monotonically increasing and the image set of this term (or function) is bounded such that $0\leq z\leq 1$ (as mentioned earlier).

Let's assume $\Psi_{t+1}^{l_t^i}$ to be a random number drawn from a continuous uniform distribution out of the interval between 0 and 1 for individual $l\in L$, $L\in\Gamma$ (that is at platform $i\in P$ at time $t\in T$) at time $t+1\in T$. The switching function W for this individual is then defined as (whereas h is again a classical Heaviside step function):

$$(8) \qquad W_{t+1}^{l_t^i}=h_{t+1}^{l_t^i}(w_{t+1}^{l_t^i}-\Psi_{t+1}^{l_t^i}).$$

This switching function allows transferring the probability that an individual switches to the other platform into a clear choice of switching: If $w_{t+1}^{l_t^i}-\Psi_{t+1}^{l_t^i}$ is greater then 0 (which is more

[3] Because of the assumption made in equation (3), the utility difference is independent of the prices set by the firms in this model.

likely if $w_{t+1}^{l_t^i}$ is relatively big) then $W_{t+1}^{l_t^i}$ becomes 1, which means that the individual is switching at the beginning of time $t+1 \in T$. If however $w_{t+1}^{l_t^i} - \Psi_{t+1}^{l_t^i}$ is smaller then 0 (which is more likely if $w_{t+1}^{l_t^i}$ is relatively small) then $W_{t+1}^{l_t^i}$ becomes 0, which means that the individual remains at his or her platform at the beginning of time $t+1 \in T$.

Number of individuals

As defined in equation (2), $N^{L_{t+1}^i}$ denotes the number of individuals $l \in L$ of a group $L \in \Gamma$ that stay at platform $i \in P$ at time $t+1 \in T$. However, because of the introduction of the switching function W, $N^{L_{t+1}^i}$ can also be defined as a recursive function (whereas $l_t^i \in L_t^i$, $l_t^j \in L_t^j$, $L \in \Gamma$, $i, j \in P$, $i \neq j$, $t, t+1 \in T$):

$$(9) \qquad N(L_{t+1}^i) = \left|L_{t+1}^i\right| = N(L_t^i) + \sum_{l_t^j} W_{t+1}^{l_t^j} - \sum_{l_t^i} W_{t+1}^{l_t^i}.$$

With this equation the model is closed: Each individual $l \in L$, $L \in \Gamma$ that is at platform $i \in P$ at time $t \in T$ calculates two net-utilities for the two platforms at time $t \in T$ according to equations (3) and (4). Further on, at the beginning of time $t+1 \in T$ each individual calculates a probability of switching to the other platform according to equation (7). Equation (8) helps to transfer this probability into a clear choice of switching. Finally equation (9) gives the new values of $N^{L_{t+1}^i}$ at the beginning of time $t+1 \in T$ (in a recursive way) so that individuals can calculate their two net-utilities again at time $t+1 \in T$ and so on. The platforms are passive in this model according to equation (5).

PARAMETERS AND PLANNED SIMULATIONS

The general agent-based model presented above seems to be a promising model to answer the question under which starting conditions two-sided markets tend to a winner-takes-all-situation and under which starting conditions both platforms can stay within the market in the long run. It is planned to transfer this general agent-based model into the NetLogo environment (Wilensky 1999), where the outcomes of the model should be received through computerised simulations, given different starting conditions (respectively parameters).

Starting conditions (or parameters) that have to be defined (and that can be varied) can be divided into four groups: Firstly, the initial number of individuals on each side ($|L|$, $|M|$ whereas $L, M \in \Gamma$ and $L \neq M$) have to be defined (and can be varied) as well as the absolute fractions of individuals on both sides that stay at each of the two platforms initially ($\left|L_0^i\right|$, $\left|L_0^j\right|$, $\left|M_0^i\right|$, $\left|M_0^j\right|$ whereas $\left|L_0^i\right| + \left|L_0^j\right| = |L|$, $\left|M_0^i\right| + \left|M_0^j\right| = |M|$, $L, M \in \Gamma$, $L \neq M$, $i, j \in P$ and $i \neq j$).

Secondly, the 'dullness term' (respectively the 'dullness function') $z(\textit{utility difference})$ has to be defined. As mentioned earlier, a lot of curve progressions of that function are possible, as long as $z(\textit{utility difference})$ is defined as a monotonically increasing function with an image set that is bounded such that $0 \leq z \leq 1$. One meaningful possibility (among many others) would be defining $z(\textit{utility difference}) = z(\Delta u)$ as a convex function capped by 1 (whereas e is Euler's number and y has to be defined reasonably):

$$(10) \qquad z(\Delta u)=\begin{cases} 0 & if \quad \Delta u \le 0 \\ e^{(\Delta u - y)} & if \quad 0<\Delta u \ and \ e^{(\Delta u - y)}<1 \\ 1 & if \quad 0<\Delta u \ and \ 1\le e^{(\Delta u - y)} \end{cases}.$$

Thirdly, the utility-weighting-factors for all individuals $\alpha^{l,M}$ (whereas $l \in L$, $L,M \in \Gamma$) have to be defined and should be varied. The systematic variation of these utility-weighting-factors for the individuals is the key to answer the research question, when – just because of the dynamics of the network-effects – two-sided markets tend to a winner-takes-all-situation and when both platforms can survive within the market. Basically, it seems reasonable (among other possibilities) to assign real numbers out of the range between -1 and 1 to all same-side utility-weighting-factors for all individuals $\alpha^{l,L}$ (whereas $l \in L$, $L \in \Gamma$) and to assign real numbers out of the range between 0 and 1 to all cross-side utility-weighting-factors for all individuals $\alpha^{l,M}$ (whereas $l \in L$, $L,M \in \Gamma$, $L \neq M$). The reason why only positive values are assigned to cross-side utility-weighting-factors is that negative values would in some sense contradict the definition of a two-sided (and multi-sided) market, which says that two-sided or multi-sided markets tie together two or more distinct groups of users in a network, whereas the individuals of each group are interested in interacting with the individuals of the other group.[4] Therefore if negative numbers were assigned to the cross-side utility-weighting-factors, the individuals of each group wouldn't be interested in interacting with the individuals of the other group.

To be more specific, it seems meaningful (among many other reasonable possibilities) to draw the same-side utility-weighting-factors for all individuals $\alpha^{l,L}$ for a distinct group $L \in \Gamma$ (whereas $l \in L$) from a defined continuous uniform distribution out of several, for example out of five defined sub-ranges (out of five possibilities), e.g. out of the ranges $[-1.0,-0.6]$, $[-0.6,-0.2]$, $[-0.2,0.2]$, $[0.2,0.6]$ and $[0.6,1.0]$.

In the same way, it seems reasonable to draw the cross-side utility-weighting-factors for all individuals $\alpha^{l,M}$ for a distinct group $L \in \Gamma$ (whereas $l \in L$, $L,M \in \Gamma$, $L \neq M$) from a defined continuous uniform distribution out of several, for example out of two defined sub-ranges (respectively possibilities), e.g. out of the ranges $[0.2,0.6]$ and $[0.6,1.0]$.

Given the two groups (respectively market sides) $L,M \in \Gamma$ (whereas $L \neq M$) and given that the same-side and cross-side utility-weighting-factors are assigned to the individuals of each group according to the defined possibilities above, to answer the described research question this setting would yield $2 \times 2 \times 5 \times 5 = 100$ possible utility-weighting-factors-combinations for each combination of the other starting conditions (respectively parameters) that have to be varied in the simulations.

Finally, the number of time periods $|T|$, the numerous simulations with the different configurations of the parameters will run until they stop, has to be defined. Since the research question refers to the outcomes of the model in the long run, it seems meaningful to define $|T|$ such that $|T| \ge 1000$.

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4 It has to be mentioned that this is not generally true for every two-sided or multi-sided market one could think of. For example, imagine a media platform (e.g. a television program) where the platform brings together viewers and advertisers. In such a case cross-side network effects and therefore cross-side utility-weighting factors may be positive in just one direction and zero or negative in the other direction and yet it is a two-sided market (Peitz/Valletti 2005; Reisinger 2004).

CONCLUSION AND OUTLOOK

Although no computerised simulations have been done so far in a simulation-environment like NetLogo, the presented general formulation of an agent-based model of a two-sided market seems very promising to study and analyze under which starting conditions (given a duopoly) two-sided and multi-sided markets tend (just because of the dynamics of the network-effects) to a winner-takes-all-situation and under which starting conditions both platforms can survive within the market. The use of an agent-based approach seems to be an appropriate and useful method in this field, since traditional approaches often have to face the problem of an exploding complexity of the created models.

Nevertheless, the model presented in this paper is relatively simple: Firstly, only two-sided markets are considered. Secondly, platforms have to be more or less passive in the model and thirdly, just one platform can be chosen from each agent at any time – which means that 'multi-homing' is not possible. In future versions of the model it is planned to weaken these limitations, especially the passivity of the platforms.

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